Eur. Phys. J. A **8**, 15–17 (2000)

THE EUROPEAN PHYSICAL JOURNAL A

Short note

A polarization relation and the measurement of the longitudinal response in pseudoscalar meson electroproduction off the nucleon

H. Schmieden[a] and L. Tiator

Institut für Kernphysik, Universität Mainz, 55099 Mainz, Germany

Received: 30 March 2000
Communicated by W. Weise

Abstract. For pseudoscalar meson electroproduction off the nucleon in parallel kinematics a relation between three polarization observables is derived. It is shown that, without Rosenbluth separation, a measurement of the longitudinal strength can be achieved through three different ways. They are discussed with preliminary MAMI data for the $p(\vec{e}, e'\vec{p})\pi^0$ reaction in the energy range of the $\Delta(1232)$ resonance.

PACS. 13.60.Le Meson production – 14.20.Gk Baryon resonance with $S = 0$ – 13.40.-f Electromagnetic processes and properties – 13.60.-r Photon and charged-lepton interactions with hadrons

1 Introduction

In the past few years, elastic and quasielastic scattering experiments yielded decisive results concerning the electromagnetic structure of proton [1] and neutron [2–6] by taking advantage of double polarization observables. These provide also high sensitivity to the longitudinal electromagnetic coupling of nucleon resonances [7]. Experimentally, these short lived resonances can be tagged through their decay into the nucleon-meson channel. Pseudoscalar meson production is thus of particular interest for resonance physics. Due to the inherent problem of separation of non-resonant background, unpolarized measurements are valuably complemented by polarization experiments, which also profit from their insensitivity to major sources of systematic uncertainties.

Furthermore, polarization observables can be utilized for a separation of longitudinal and transverse response. Thus, the experimental difficulties of the standard Rosenbluth-technique [8,9] can be circumvented. Three different methods for the extraction of the ratio of longitudinal and transverse strength are discussed on the basis of first, preliminary $p(\vec{e}, e'\vec{p})\pi^0$ data in the energy range of the $\Delta(1232)$ resonance [10,11] from the Mainz Microtron MAMI.

[a] e-mail: hs@kph.uni-mainz.de

2 Recoil polarization in parallel kinematics

The present double-polarization experiments focus on the situation of parallel kinematics, where the recoiling nucleon of the $p(e, e'p)\pi^0$ reaction is detected along the direction of the momentum transfer, $\vec{q}$. In this case the components of the recoil nucleon polarization [1] are given by

$$P_x = P_e \cdot c_- \cdot \frac{R^t_{LT'}}{R_T + \epsilon_L R_L}, \tag{1}$$

$$P_y = c_+ \cdot \frac{R^n_{LT}}{R_T + \epsilon_L R_L}, \tag{2}$$

$$P_z = P_e \cdot c_0 \cdot \frac{R^l_{TT'}}{R_T + \epsilon_L R_L}. \tag{3}$$

The coordinate frame is defined relative to the electron scattering plane as depicted in fig.1. In contrast to ref.[7] here we use the notation of Drechsel and Tiator [12]. The structure functions, R^i_K, have to be taken at the pion cm-angle of $\Theta^{\rm cm}_\pi = 180°$. P_e denotes the longitudinal electron polarization, and the kinematical factors are

$$c_\pm = \sqrt{2\epsilon_L(1 \pm \epsilon)} \quad \text{and} \quad c_0 = \sqrt{1 - \epsilon^2}, \tag{4}$$

where $\epsilon = (1 + 2\vec{q}^{\,2}/Q^2 \tan^2 \frac{1}{2}\vartheta_e)^{-1}$ and $\epsilon_L = (Q^2/\omega^2_{\rm cm})\epsilon$ represent the degrees of tranverse and longitudinal polarization of the virtual photon, respectively. Q^2 is the

[1] Target polarization is equivalent to the measurement of recoil polarization when the cross-section asymmetry with regard to the reversal of beam helicity is considered.

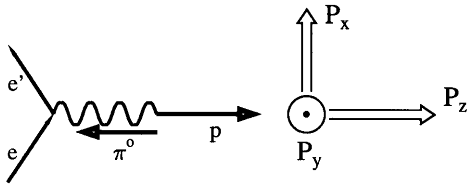

Fig. 1. Electroproduction of pseudoscalar mesons in parallel kinematics for the example of the $p(\vec{e}, e'p)\pi^0$ reaction. The components $P_{x,y,z}$ of the proton polarization are defined relative to the electron scattering plane.

negative squared four-momentum transfer, ϑ_e the electron scattering angle and ω_{cm} the energy transfer in the photon-nucleon cm frame.

From the components of recoil polarization (eqs. (1-3)) we define reduced polarizations through normalization by the virtual photon polarization factors:

$$\chi_x = \frac{1}{P_e c_-} \cdot P_x = \frac{R^t_{LT'}}{R_T + \epsilon_L R_L}, \quad (5)$$

$$\chi_y = \frac{1}{c_+} \cdot P_y = \frac{R^n_{LT}}{R_T + \epsilon_L R_L}, \quad (6)$$

$$\chi_z = \frac{1}{P_e c_0} \cdot P_z = \frac{R^l_{TT'}}{R_T + \epsilon_L R_L}. \quad (7)$$

The structure functions are conveniently expressed in terms of six helicity amplitudes, $H_{1\text{-}6}$, or CGLN amplitudes, $F_{1\text{-}6}$ [12–14]. This yields:

$$R_T = \frac{1}{2}|H_4|^2 = |F_1 + F_2|^2, \quad (8)$$

$$R_L = |H_6|^2 = |F_5 - F_6|^2, \quad (9)$$

$$R^t_{LT'} = -\frac{1}{\sqrt{2}}\Re e\{H_6^* H_4\} = -\Re e\{(F_6^* - F_5^*)(F_1 + F_2)\}, \quad (10)$$

$$R^n_{LT} = \frac{1}{\sqrt{2}}\Im m\{H_6^* H_4\} = \Im m\{(F_6^* - F_5^*)(F_1 + F_2)\}, \quad (11)$$

$$R^l_{TT'} = \frac{1}{2}|H_4|^2 = |F_1 + F_2|^2, \quad (12)$$

$R^t_{LT'}$ and R^n_{LT} represent real and imaginary parts of the same complex interference term. The equality

$$R_T = R^l_{TT'} \quad (13)$$

is a pecularity of parallel kinematics.

3 Polarization relation and L/T separation

From eqs. (8-12) it can be easily seen that

$$\chi_x^2 + \chi_y^2 = \frac{\frac{1}{2}|H_4|^2|H_6|^2}{(R_T + \epsilon_L R_L)^2} = \frac{R_T \cdot R_L}{(R_T + \epsilon_L R_L)^2} \quad (14)$$

and

$$\chi_z^2 = \frac{R_T^2}{(R_T + \epsilon_L R_L)^2}. \quad (15)$$

Therefore, the ratio between longitudinal and transverse response is given by

$$\frac{R_L}{R_T} = \frac{\chi_x^2 + \chi_y^2}{\chi_z^2}. \quad (16)$$

The extraction of R_L/R_T from eq. (16) requires the measurement of all three polarization components. It can also be obtained from χ_z alone [15] by rewriting eq. (15):

$$\frac{R_L}{R_T} = \frac{1}{\epsilon_L}\left(\frac{1}{\chi_z} - 1\right). \quad (17)$$

Combining eqs. (17) and (16) we directly receive a model-independent relation between the three reduced polarizations:

$$\chi_x^2 + \chi_y^2 = \frac{1}{\epsilon_L}\chi_z(1 - \chi_z). \quad (18)$$

This equation relates the absolute value of the transverse polarization with the longitudinal polarization. It represents a constraint for any model. For example, it is perfectly fullfilled by the Mainz Unitary Isobar Model [16] for pion photo and electroproduction.

The relation can also serve as a consistency check for experimental data. Up to now there is one preliminary data set available with all three components of proton polarization simultaneously measured in the reaction $p(\vec{e}, e'\vec{p})\pi^0$ at the energy of the Δ-resonance [10,11]. Within their present errors these data fullfill eq. (18):

$$\chi_x^2 + \chi_y^2 = 0.0348 \pm 0.0045_{\text{stat}} \pm 0.0031_{\text{syst}} = \frac{1}{\epsilon_L}\chi_z(1 - \chi_z) = 0.0519 \pm 0.022_{\text{stat}} \pm 0.007_{\text{syst}}. \quad (19)$$

We note, however, that the rhs of eq. (19) does not produce a very strong constraint. This is due to the unfavorable error propagation when the small deviation of χ_z from unity is measured.

The determination of R_L/R_T from eq. (17) suffers by the same reason. We obtain

$$\frac{R_L}{R_T} = 0.066^{+0.038}_{-0.034}{}_{\text{stat}} \pm 0.011_{\text{syst}}. \quad (20)$$

A relative statistical error of 6.2 % in χ_z is amplified to 57.6 % in R_L/R_T. This is a consequence of the weak influence of R_L on the longitudinal polarization component. Therefore, in the $\Delta(1232)$ region P_z should rather be used as an experimental consistency check, because it is almost entirely determined by the polarization of the electron beam and electron kinematics [7]:

$$P_z \simeq P_e \cdot c_0 \quad (21)$$

A better way to determine the longitudinal strength is through eq. (16), which yields

$$\frac{R_L}{R_T} = 0.044 \pm 0.006_{\text{stat}} \pm 0.004_{\text{syst}}. \qquad (22)$$

However, all three polarization components need to be measured simultaneously. A procedure which requires only the two transverse reduced polarizations starts from eq. (14) as a quadratic equation for R_L/R_T. The solutions are

$$\frac{R_L}{R_T} = \left[\alpha_L \pm \sqrt{\alpha_L^2 - \epsilon_L^2}\right]^{-1}, \qquad (23)$$

where

$$\alpha_L = \frac{1}{2(\chi_x^2 + \chi_y^2)} - \epsilon_L. \qquad (24)$$

To the extent that $R_L/R_T \ll 1/\epsilon_L$, eq. (23) can be simplified to

$$\frac{R_L}{R_T} = \frac{1}{2\alpha_L}. \qquad (25)$$

The second solution yields $R_L > R_T$ and is thus obviously unphysical. With eq. (25) the preliminary MAMI data [10] yield the result

$$\frac{R_L}{R_T} = 0.040 \pm 0.006_{\text{stat}} \pm 0.004_{\text{syst}}. \qquad (26)$$

For the latter two approaches it is possible to practically maintain the experimental errors of the reduced polarizations in the extracted ratio. In contrast to the extraction through eq. (17) there is no error amplification, but the measurement of two or three polarization components is required. Experimentally, the false systematic asymmetries of the recoil polarimeter need to be under control. While they can be eliminated in the electron-helicity dependent components P_x and P_z, they influence the extraction of P_y.

4 Summary and conclusions

For the case of electroproduction of pseudoscalar mesons off the nucleon in parallel kinematics with longitudinally polarized beam and with either measurement of recoil nucleon polarization or target polarization a set of three reduced polarizations has been defined. There exists a model-independent relation between the quadratic sum of the two transverse reduced polarizations and the longitudinal one. It puts a constraint on both phenomenological models and experimental data.

The polarization observables offer the possibility to measure the ratio of longitudinal to transverse strength without the need of a Rosenbluth-separation. Three different ways have been discussed which require the measurement of one, two or three of the reduced polarizations. While the L/T-ratio from χ_z alone suffers from a very unfavourable error propagation, in particular the quadratic sum $\chi_x^2 + \chi_y^2$ is well suited for a measurement of the longitudinal response.

The authors are indebted to Thomas Pospischil for sharing preliminary experimental results of his doctoral thesis prior to publication. This work was supported by the Deutsche Forschungsgemeinschaft (SFB 443).

References

1. M.K. Jones et al., Phys. Rev. Lett. **84**, 1398 (2000).
2. T. Eden et al., Phys. Rev. C **50**, R1749 (1994).
3. M. Ostrick et al., Phys. Rev. Lett. **83**, 276 (1999).
4. C. Herberg et al., Eur. Phys. J. A **5**, 131 (1999).
5. I. Passchier et al., Phys. Rev. Lett. **82**, 4988 (1999).
6. D. Rohe et al., Phys. Rev. Lett. **83**, 4257 (1999).
7. H. Schmieden, Eur. Phys. J. A **1**, 427 (1998).
8. K.I. Blomqvist et al., Z. Phys. A **353**, 415 (1996) and A. Liesenfeld et al., Phys. Lett. B **468**, 20 (1999).
9. M.O. Distler et al., Phys. Rev. Lett. **80**, 2294 (1998).
10. H. Schmieden, *Proceedings of the 15th International Conference on Particles and Nuclei, Uppsala 1999*, Nucl. Phys. A **663 & 664**, 21c (2000).
11. Th. Pospischil, doctoral thesis, Mainz, in preparation.
12. D. Drechsel and L. Tiator, J. Phys. G: Nucl. Part. Phys. **18**, 449 (1992).
13. G. Knöchlein, D. Drechsel and L. Tiator, Z. Phys. A **352**, 327 (1995).
14. G.F. Chew et al., Phys. Rev. **106**, 1345 (1957).
15. J.J. Kelly, Phys. Rev. C **60**, 054611 (1999).
16. D. Drechsel, O. Hanstein, S.S. Kamalov and L. Tiator, Nucl. Phys. A **645**, 145 (1999).